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WAVEGUIDE TRANSMISSION CAVITY WITH LOSSES

February 1966

J. L. Altman

Prepared for

DIRECTORATE OF RADAR AND OPTICS
ELECTRONIC SYSTEMS DIVISION
AIR FORCE SYSTEMS COMMAND
UNITED STATES AIR FORCE
L. G. Hanscom Field, Bedford, Massachusetts

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Project 750
Prepared by

THE MITRE CORPORATION
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ABSTRACT

The symmetrical – but lossy – waveguide transmission cavity is analyzed with respect to resonant length, minimum and maximum insertion losses, and 3-db frequencies (loaded Q-factor). Losses are considered to originate both in the cavity proper and in the input and output discontinuities. A preferred representation of those discontinuities makes it possible effectively to lump all the various losses together. All the quantities of interest (resonant length, insertion losses and Q_L-factor) are expressed in terms of the input and output discontinuity equivalent circuit. The results are quite general, but they will be of particular interest in the case of a transmission filter whose input and output are voltage-controlled varactors.

Normalized plots of resonant lengths, insertion loss at resonance, and Q_L-factors conclude the study. The reactive component of the input-output equivalent circuit is the independent variable. The resistive component is the parameter. Loci of constant insertion losses, plotted on the Q_L graphs, makes it possible to determine the maximum achievable Q_L for a given insertion loss and resistive component; or the maximum resistive component for a given Q_L and a prescribed insertion loss; or the minimum insertion loss for given Q_L and resistive component.

REVIEW AND APPROVAL

This technical report has been reviewed and is approved.

HARRY N. BYRAM
ESD Project Officer
Project 750

TABLE OF CONTENTS

SECTION I

INTRODUCTION

The symmetrical – but lossy – waveguide transmission cavity is analyzed with regard to resonant length, minimum and maximum insertion losses, and 3-db frequencies (loaded Q-factor). Losses are considered to originate both in the cavity proper and in the input and output two-ports. All the quantities enumerated above are obtained in terms of the two-port equivalent circuit. Although the results are general, they are of particular interest in the case of a transmission filter whose input and output two-ports are voltage-controlled varactors.

The cavity is treated as a length of line inserted between two symmetrical discontinuities (the input and output two-ports). A preferred representation [Equation (10)] of the discontinuities enables one to compute the effects of losses on the resonant length, and to effectively lump them with the waveguide attenuation [Equations (17) and ff.]. The resonant length is obtained for the condition of maximum transmission [Equation (24)], and both the values of maximum and minimum transmissions are determined [Equations (23) and (26)]. In a similar fashion, the electrical lengths that reduce the transmission by 3 db below transmission at resonance are obtained in Equation (28). Finally, those "3-db" lengths are related to frequencies, and an expression for Q_L is derived in Equation (37). Representative plots of cavity length, insertion loss and Q_L-factors are discussed in Section V.

SECTION II

PREFERRED REPRESENTATION OF A LOSSY TWO-PORT

Let the input or output discontinuity be represented by a shunt impedance $\bar{z}_e$, shown in Figure 1.* The scattering matrix of this shunt impedance is

$$[S] = \frac{1}{1 + 2\bar{z}_e} \begin{vmatrix} -1 & 2\bar{z}_e \\ 2\bar{z}_e & -1 \end{vmatrix}, \qquad (1)$$

regardless of whether $\bar{z}_e$ is real, imaginary, or complex.

Write $\bar{z}_e$ in the form

$$\bar{z}_e = \bar{r}_e + j\bar{x}_e = j\left|\bar{z}_e\right| e^{-j\psi}, \qquad (2)$$

where

$$\psi = \tan^{-1} \frac{\bar{r}_e}{\bar{x}_e} \qquad (3)$$

will be

(a) less than $\pi/2$ if $\bar{x}_e$ is positive, and

(b) more than $\pi/2$ and less than π if $\bar{x}_e$ is negative (see Figure 2).

---

*Any waveguide discontinuity may be represented by a simple shunt impedance. If the discontinuity is "thin," then the equivalent circuit applies at the exact plane of discontinuity. If the discontinuity is "thick," then it applies at some other plane whose location is dependent on the thickness.

With this notation, the scattering matrix becomes

$$[S] = \frac{1}{1 + j2\left|\bar{z}_e\right| e^{-j\psi}} \begin{vmatrix} -1 & j2\left|\bar{z}_e\right| e^{-j\psi} \\ j2\left|\bar{z}_e\right| e^{-j\psi} & -1 \end{vmatrix} . \qquad (1')$$

Whereas the two reference planes of the two-port shown in Figure 1 coincide at the plane of discontinuity or "thereabout,"* it will prove advantageous to move each reference plane by an electrical angle φ in such a way that

$$s_{11} = s_{22} = \text{real and negative, or}$$

$$[S'] = e^{j2\varphi}[S] . \qquad (4)$$

with

$$\frac{e^{j2\varphi}}{1 + j2\left|\bar{z}_e\right| e^{-j2\psi}} = \text{real and positive.}$$

This will be the case if

$$2\varphi = \tan^{-1} \frac{2\bar{X}_e}{1 + 2\bar{r}_e} . \qquad (5)$$

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* See footnote on page 2.

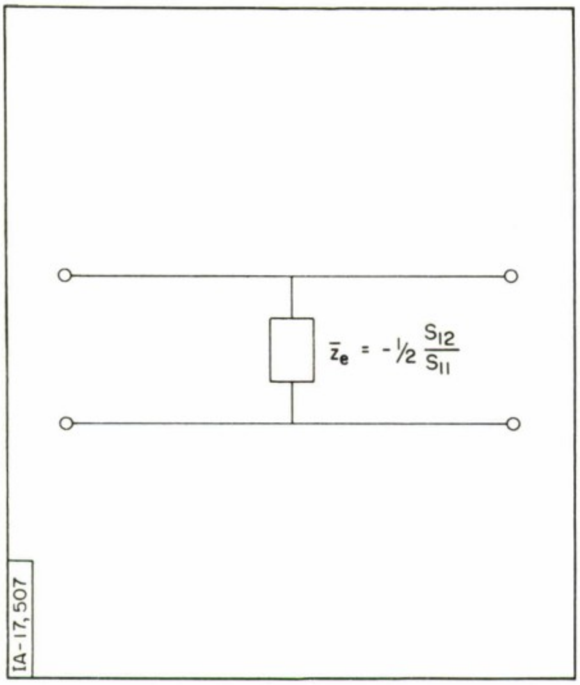

Figure 1. Equivalent Circuit of Input or Output

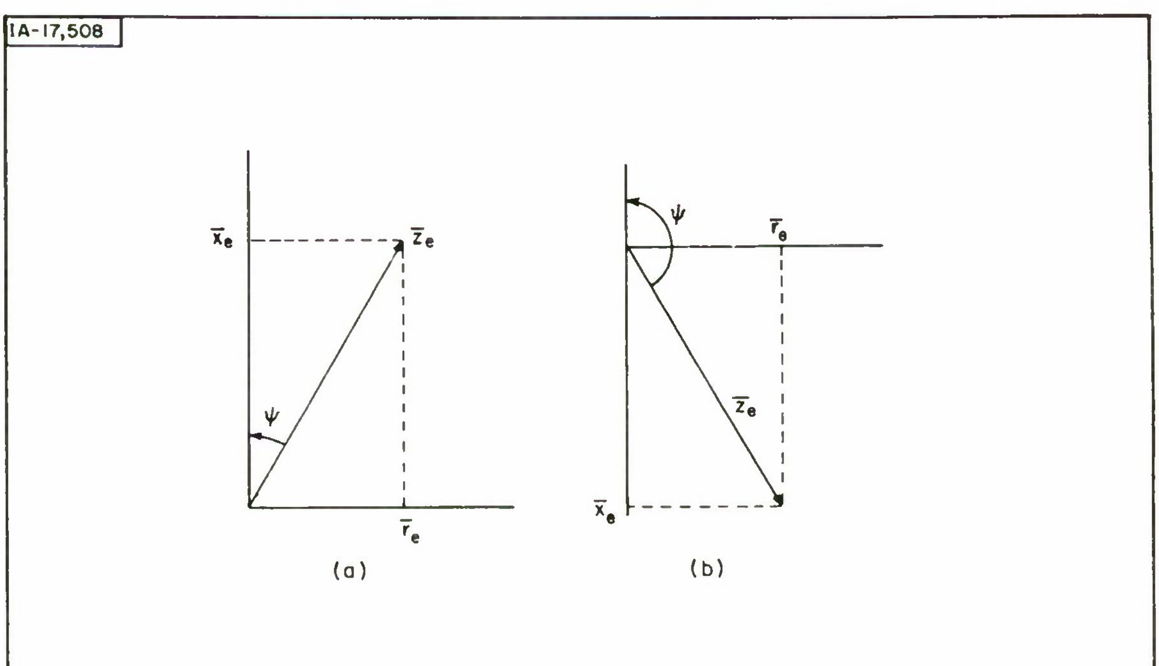

Figure 2. (a) Inductive Impedance (b) Capacitive Impedance

so that now

$$\frac{e^{j2\varphi}}{1+j2\left|\bar{z}_e\right| e^{-j\psi}} = \frac{1}{\left[\left(1+2\bar{r}_e\right)^2+4\bar{X}_e^{\,2}\right]^{1/2}}$$

$$= \frac{1}{\left[1+\left|2\bar{z}_e\right|^2+4\bar{r}_e\right]^{1/2}} = \frac{1}{\left[1+\left|2\bar{z}_e\right|^2\right]^{1/2}} \cdot \frac{1}{\left(1+\dfrac{4\bar{r}_e}{1+\left|2\bar{z}_e\right|^2}\right)^{1/2}} \cdot$$

If $\left(1+\left|2\bar{z}_e\right|^2\right) > 4\bar{r}_e$, that is, if losses are not unduly large,* then Equation (6) becomes, approximately,

$$\frac{e^{j2\varphi}}{1+j2\left|\bar{z}_e\right| e^{-j\psi}} \approx \frac{1}{\left[1+\left|2\bar{z}_e\right|^2\right]^{1/2}}\, e^{-\alpha_p} , \tag{7}$$

where

$$\alpha_p = \frac{2\bar{r}_e}{1+\left|2\bar{z}_e\right|^2} . \tag{8}$$

Now define the coupling factor k as

$$k = \frac{2\left|\bar{z}_e\right|}{\left[1+\left|2\bar{z}_e\right|^2\right]^{1/2}} . \tag{9}$$

---

*Equation (7) will hold within a few percent, regardless of $\bar{x}_e$, provided that $r_e < 0.1$.

The substitution of Equations (7) and (9) into Equation (4) results in the desired expression for [S] :

$$[S'] = e^{-\alpha p} \begin{vmatrix} -\sqrt{1-k^2} & jke^{-j\psi} \\ jke^{-j\psi} & -\sqrt{1-k^2} \end{vmatrix} . \tag{10}$$

In summary, [S'] is the scattering matrix for the input or output two-port when the reference planes are chosen such that $s_{11} = s_{22}$ is real and negative. The effects of losses are explicitly brought out in the terms $e^{-\alpha p}$ and $e^{-j\psi}$. This is the preferred representation which will be used to compute the resonance conditions and the Q-factors.

SECTION III

RESONANCE CONDITIONS OF A WAVEGUIDE TRANSMISSION CAVITY WITH LOSSY ELEMENTS

A waveguide transmission cavity is depicted in Figure 3. The input and output extend to the planes where, for the respective two-ports, s_{11} and s_{22} are real and negative. The two-ports are characterized by k, α_p and ψ, and (implicitly) by φ. The cavity proper is the length of line inserted between the two-ports. It is characterized by its electrical length θ and a one-way attenuation factor α_ℓ expressed in nepers.

Conditions at the input two-port are obtained from the relations

$$\vec{b} = [S]\,\vec{a},$$

which become, upon substituting the preferred expression of Equation (10),

$$-e^{-\alpha_p}\sqrt{1-k^2}\,a_1 + jke^{-(\alpha_p + j\psi)}\,a_2 = b_1, \tag{11a}$$

$$jke^{-(\alpha_p + j\psi)}\,a_1 - e^{-\alpha_p}\sqrt{1-k^2}\,a_2 = b_2. \tag{11b}$$

Conditions at the output two-port are

$$-e^{-\alpha_p}\sqrt{1-k^2}\,a_1' = b_1', \tag{12a}$$

$$jke^{-(\alpha_p + j\psi)}\,a_1' = b_2'. \tag{12b}$$

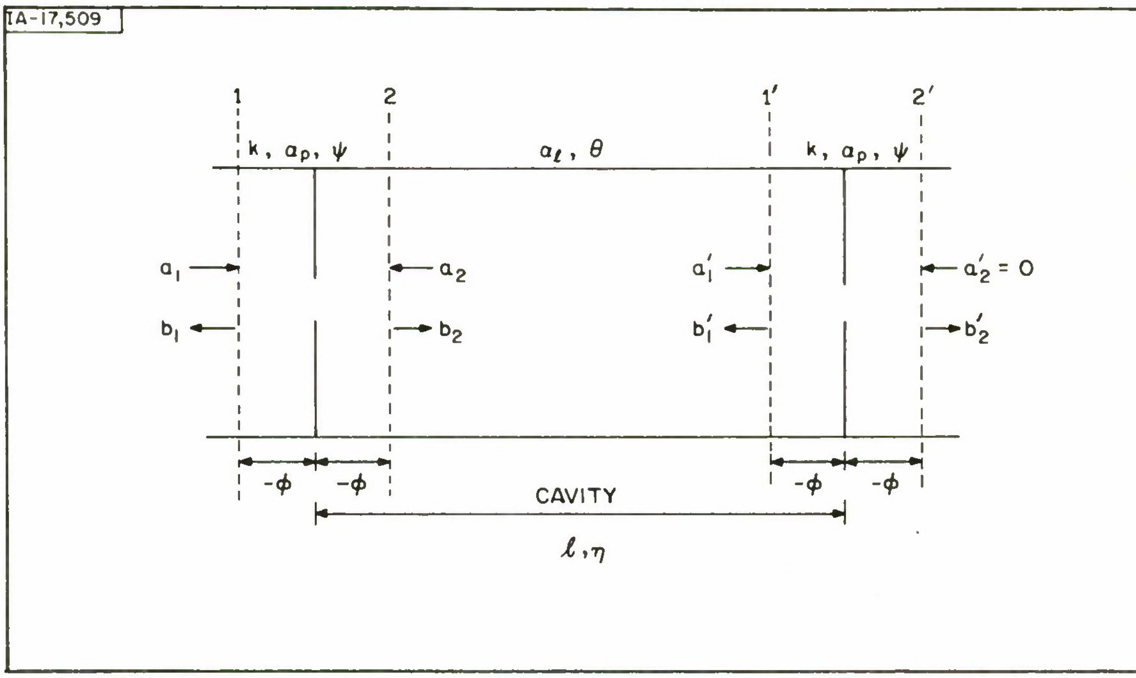

Figure 3. Waveguide Transmission Cavity with Lossy Elements

But a_2 is related to b_1', and a_1' is related to b_2 by

$$a_2 = b_1' e^{-\left(\alpha_\ell + j\theta\right)} , \tag{13a}$$

$$a_1' = b_2 e^{-\left(\alpha_\ell + j\theta\right)} . \tag{13b}$$

Thus, there are six unknown quantities: a_2, a_1', b_1, b_2, b_1' and b_2', but six independent linear equations. It is then possible to solve for any of the unknown in terms of the circuit parameters and of the input a_1. By substituting Equation (13a) into Equations (11), and Equation (13b) into Equations (12), we obtain

$$b_1 = -e^{-\alpha_p}\sqrt{1-k^2}\,a_1 + jke^{-\left(\alpha_p + j\psi\right)}e^{-\left(\alpha_\ell + j\theta\right)}b_1' \, , \tag{14a}$$

$$b_2 = jke^{-\left(\alpha_p + j\psi\right)}a_1 - e^{-\alpha_p}\sqrt{1-k^2}\,e^{-\left(\alpha_\ell + j\theta\right)}b_1' \; ; \tag{14b}$$

and

$$b_1' = -e^{-\alpha_p}\sqrt{1-k^2}\,e^{-\left(\alpha_\ell + j\theta\right)}b_2 \, . \tag{15a}$$

$$b_2' = jke^{-\left(\alpha_p + j\psi\right)}e^{-\left(\alpha_\ell + j\theta\right)}b_2 \, . \tag{15b}$$

One may eliminate b_1' between Equations (14a) and (15a) and obtain, after simplifications,

$$b_2 = \frac{jk\,e^{-\left(\alpha_p + j\psi\right)}}{1 - \left(1 - k^2\right)e^{-2\left(\alpha_p + \alpha_\ell + j\theta\right)}}\,a_1 \, . \tag{16}$$

By successive eliminations, one obtains

$$a_1' = \frac{-jk\,e^{-\left[\alpha_p + \alpha_\ell + j(\theta + \psi)\right]}}{1 - \left(1 - k^2\right)e^{-2\left(\alpha_p + \alpha_\ell + j\theta\right)}}\,a_1 \; ; \tag{17}$$

$$b_1' = \frac{-jk\sqrt{1-k^2}\,e^{-\left[2\alpha_p + \alpha_\ell + j(\theta + \psi)\right]}}{1 - \left(1 - k^2\right)e^{-2\left(\alpha_p + \alpha_\ell + j\theta\right)}}\,a_1 \; ; \tag{18}$$

$$a_2 = \frac{-jk\sqrt{1-k^2}\, e^{-\left[2\alpha_p + 2\alpha_\ell + j(2\theta + \psi)\right]}}{1 - \left(1-k^2\right) e^{-2\left(\alpha_p + \alpha_\ell + j\theta\right)}}\, a_1 \;; \tag{19}$$

$$b_2' = \frac{-k^2 e^{-\left[2\left(\alpha_p + j\psi\right) + \left(\alpha_\ell + j\theta\right)\right]}}{1 - \left(1-k^2\right) e^{-2\left(\alpha_p + \alpha_\ell + j\theta\right)}}\, a_1 \;; \tag{20}$$

$$b_1 = -e^{-\alpha_p}\sqrt{1-k^2} + \frac{k^2\sqrt{1-k^2}\, e^{-\left[3\alpha_p + 2\alpha_\ell + j2(\theta + \psi)\right]}}{1 - \left(1-k^2\right) e^{-2\left(\alpha_p + \alpha_\ell + j\theta\right)}} \;. \tag{21}$$

All the quantities will be maximum in magnitude when

$$\theta = n\pi\,, \quad n = 1, 2, 3, \ldots \tag{22}$$

except for b_1, the input reflection, which will be minimum in magnitude. Equation (22) expresses the <u>resonance condition</u>. At resonance, the magnitude of b_2' (which is proportional to the square root of the transmitted power) becomes

$$\boxed{\left|b_2'\right|_{max} = \frac{k^2 e^{-\left[2\alpha_p + \alpha_\ell\right]}}{1 - \left(1-k^2\right) e^{-2\left(\alpha_p + \alpha_\ell\right)}}\, \left|a_1\right|} \;. \tag{23}$$

Were it not for the loss mechanisms $\left(\alpha_p \text{ and } \alpha_\ell\right)$, $\left|b_2'\right|$ would be equal to $\left|a_1\right|$ (i.e., the output power would be equal to the input power), and b_1 would be zero (i.e., no input reflection).

In summary, the resonant length of a waveguide transmission cavity with lossy elements is, by Equations (5) and (22):

$$\boxed{\eta_0 = n\pi - \tan^{-1} \frac{2\bar{x}_e}{1 + 2\bar{r}_e} , \quad n = 1, 2, 3, \ldots} \quad . \tag{24}$$

Note that the waveguide attenuation does not enter this expression and that the losses of the two-ports will perturbate η_0 but slightly if $\bar{r}_e < 1$.

Another worthwhile consideration is that of the maximum insertion loss. This will prevail when

$$\theta = n \frac{\pi}{2} , \quad n = 1, 3, 5, 7, \ldots \quad , \tag{25}$$

and corresponds to the anti-resonance condition. Then,

$$\left| b_2' \right|_{min} = \frac{k^2 e^{-\left[2\alpha_p + \alpha_\ell\right]}}{1 + \left(1 - k^2\right) e^{-2\left(\alpha_p + \alpha_\ell\right)}} \left| a_1 \right| \quad . \tag{26}$$

In the absence of dissipation, this quantity would be

$$\frac{k^2}{2 - k^2} \left| a_1 \right| \quad ,$$

and would approach zero only as k (the "coupling" coefficient of the two-ports) approaches zero.*

*This important fact becomes obscured in the commonly used RLC equivalent circuit representation.

SECTION IV

HALF-POWER POINTS AND THE Q_L-FACTOR OF THE LOSSY CAVITY

It proves convenient – although transmission never actually becomes zero even at anti-resonance – to consider the 3-db points; that is, the frequencies at which transmission drops 3 db below transmission at resonance. Thus, one may solve for θ_1 and θ_2 — the electrical lengths at those 3 db points — by letting $\left|b_2'\right|$ be $0.707 \left|b_2'\right|_{max}$, or, by Equations (20) and (23),

$$\left|1 - \left(1 - k^2\right) e^{-2\left(\alpha_p + \alpha_\ell + j\theta_{1,2}\right)}\right| = \sqrt{2}\left[1 - \left(1 - k^2\right) e^{-2\left(\alpha_p + \alpha_\ell\right)}\right];$$

that is:

$$\left[1 - \left(1 - k^2\right) e^{-2\left(\alpha_p + \alpha_\ell\right)} \cos 2\theta_{1,2}\right]^2 + \left[\left(1 - k^2\right) e^{-2\left(\alpha_p + \alpha_\ell\right)} \sin 2\theta_{1,2}\right]^2$$

$$= 2\left[1 - \left(1 - k^2\right) e^{-2\left(\alpha_p + \alpha_\ell\right)}\right]^2 . \quad (27)$$

Let

$$\left(1 - k^2\right) e^{-2\left(\alpha_p + \alpha_\ell\right)} = a, \quad 0 < a < 1 .$$

Then, by Equation (27), the 3-db point conditions correspond to

$$\left[1 - a \cos 2\theta_{1,2}\right]^2 + a^2 \sin^2 2\theta_{1,2} = 2\left[1 - a\right]^2 , \quad (27')$$

or

$$\cos 2\theta_{1,2} = \frac{a(4 - a) - 1}{2a} . \quad (28)$$

It may be verified that

$$\cos 2\theta_{1,2} = 1$$

for $a = 1$ (infinite Q's), and that

$$\cos 2\theta_{1,2} = -1$$

for $a = 0.12$ (loss and coupling such that anti-resonance transmission is barely 3 db below transmission at resonance).

In accordance with Equation (24), the total electrical length of the transmission cavity at the 3-db points is

$$\eta_{1,2} = \mp \frac{1}{2} \cos^{-1} \left[\frac{a(4-a) - 1}{2a} \right] + \tan^{-1} \frac{2\bar{x}_e}{1 + 2\bar{r}_e} + n\pi \quad , \tag{29}$$

where

the minus sign is associated with ω_1, the lower 3-db frequency,
the plus sign is associated with ω_2, the upper 3-db frequency, and
$\frac{1}{2} \cos^{-1} \left[\frac{a(4-a) - 1}{2a} \right]$ is confined to the first quadrant only.

The loaded Q-factor, Q_L, is defined in terms of the resonant frequency ω_0 and of the frequencies ω_1 and ω_2 as

$$Q_L = \frac{\omega_0}{\omega_2 - \omega_1} \quad . \tag{30}$$

In order to evaluate Q_L in terms of the circuit parameters, one must introduce the quantities η_0 [Equation (24)], and η_1 and η_2 [Equation (29)].

Since the actual cavity length ℓ (meters) is invariant, one immediately obtains the following equations:

$$\eta_0 = \gamma_0 \ell = \frac{2\pi}{\lambda g_0} \ell \ , \tag{31a}$$

$$\eta_1 = \gamma_1 \ell = \frac{2\pi}{\lambda g_1} \ell \ , \tag{31b}$$

$$\eta_2 = \gamma_2 \ell = \frac{2\pi}{\lambda g_2} \ell \ , \tag{31c}$$

where

γ is the waveguide propagation constant,

λg is the waveguide wavelength, and

the subscripts 0, 1, 2 refer, respectively, to the resonant, lower half-power and upper half-power frequencies

Furthermore, since

$$\omega^2 = \frac{\gamma^2 + k_c^2}{\mu \epsilon} \ , \tag{32}$$

and

$$\omega = \frac{2\pi}{\sqrt{\mu \epsilon}} \frac{1}{\lambda} \ , \tag{33}$$

where

k_c is the cutoff propagation constant, and

λ is the free-space wavelength,

it may be verified that

$$d\omega = \frac{1}{\sqrt{\mu\,\epsilon}}\;\frac{\lambda}{\lambda g}\;d\gamma\;. \tag{34}$$

If the bandwidth is not too large (or if the line is nondispersive), then Equation (34) may be used as the basis for the approximation

$$\Delta\omega = \omega_2 - \omega_1 \approx \frac{1}{\sqrt{\mu\epsilon}}\;\frac{\lambda_0}{\lambda g_0}\left(\gamma_2 - \gamma_1\right)\;. \tag{34'}$$

Consequently, from Equations (30) and (33), Q_L becomes

$$Q_L = 2\pi\;\frac{\lambda g_0}{\lambda_0^2}\;\frac{1}{\left(\gamma_2 - \gamma_1\right)}\;. \tag{35}$$

By Equations (31), one can eliminate γ_1 and γ_2 to obtain

$$Q_L = 2\pi\;\frac{\lambda g_0}{\lambda_0^2}\;\frac{\ell}{\left(\eta_2 - \eta_1\right)}$$

$$= \left(\frac{\lambda g_0}{\lambda_0}\right)^2\;\frac{\eta_0}{\eta_2 - \eta_1}\;. \tag{36}$$

Hence, from Equations (24) and (29), the expression for Q_L becomes

$$Q_L = \frac{n\pi - \tan^{-1}\dfrac{2\bar{x}_e}{1 + 2\bar{r}_e}}{\cos^{-1}\left[\dfrac{a(4 - a) - 1}{2a}\right]}\cdot\left(\frac{\lambda g_0}{\lambda_0}\right)^2\;. \tag{37}$$

where

$$a = (1 - k^2)\, e^{-2(\alpha_p + \alpha_\ell)} = \frac{1}{1 + 4\left(\bar{x}_e^2 + \bar{r}_e^2\right)}\, e^{-\left\{4\bar{r}_e \Big/ \left[1 + 4\left(\bar{x}_e^2 + \bar{r}_e^2\right)\right]\right\} + 2\alpha_\ell}$$

If the bandwidth is so large that the approximation of Equation (34') is poor, then it is well-advised — following S. B. Cohn[1] — to consider the exact expression[2]

$$\frac{\eta_0}{\eta_2 - \eta_1} = \frac{1/\lambda g_0}{1/\lambda g_2 - 1/\lambda g_1} = \frac{\lambda g_1 + \lambda g_2}{2\left(\lambda g_1 - \lambda g_2\right)}$$

$$= \frac{n\pi - \tan^{-1} \dfrac{2\bar{x}_e}{1 + 2\bar{r}_e}}{\cos^{-1}\left[\dfrac{a(4 - a) - 1}{2a}\right]} \,. \tag{38}$$

Then, from the values of λg_1 and λg_2 (and the waveguide cutoff propagation constant), ω_1, ω_2 and ω_0 can be computed, and Q_L determined by Equation (30).

---

[1] S. B. Cohn, "Direct-Coupled Resonant Filters," Proc. IRE, 45, Feb. 1957, 187-196.

[2] By the substitution of $\lambda g_0 = 2\lambda g_1 \lambda g_2/(\lambda g_1 + \lambda g_2)$, which is derived in J. L. Altman, Microwave Circuits, D. Van Nostrand Co., Inc., 1964, 141.

For an analysis of the lossless waveguide transmission cavity, see ibidem, Section 5. 9, pp. 247-255.

SECTION V

GRAPHS AND CONCLUSIONS

The function

$$2\varphi = \frac{180}{\pi}\tan^{-1}\frac{2\overline{x}_e}{1+2\overline{r}_e} \text{ degrees}$$

is plotted versus $\overline{x}_e$, in Figure 4, for various values of $\overline{r}_e$.* This function expresses the <u>decremental length</u>; i.e., the length which must be subtracted from $n\pi$ [see Equation (24)]. This function will be positive for $\overline{x}_e > 0$, so that the cavity length will be less than an integral number of half-wavelengths. It will be negative for $\overline{x}_e < 0$, so that the cavity length will be <u>more</u> than an integral number of half-wavelengths. Note that the curves corresponding to $\overline{r}_e = 0$, $\overline{r}_e = 0.001$, and $\overline{r}_e = 0.01$ essentially coalesce within the line thickness so that, indeed, $\overline{r}_e$ plays a very minor role in affecting the resonant length.

Figure 5 is a plot of insertion loss (db) at resonance versus $|\overline{x}_e|$ for various values of $\overline{r}_e$. It is obtained from Equation (23) via Equation (9). Note that the insertion loss at resonance is the same, whether $\overline{x}_e > 0$ or $\overline{x}_e < 0$. The plots speak for themselves regarding the importance of $\overline{r}_e$ (recall that, for the lossless guide, the insertion loss is 0 db if $\overline{r}_e = 0$, regardless of $\overline{x}_e$).

Figure 6 is a plot of $Q_L(\lambda_0/\lambda g_0)^2$ in the case of capacitive discontinuities [$n = 1$ in Equation (24)], $\alpha_\ell = 0$, to bring out the effect of

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*I am indebted to J. Pearlman for the computation of the data from which the curves were plotted.

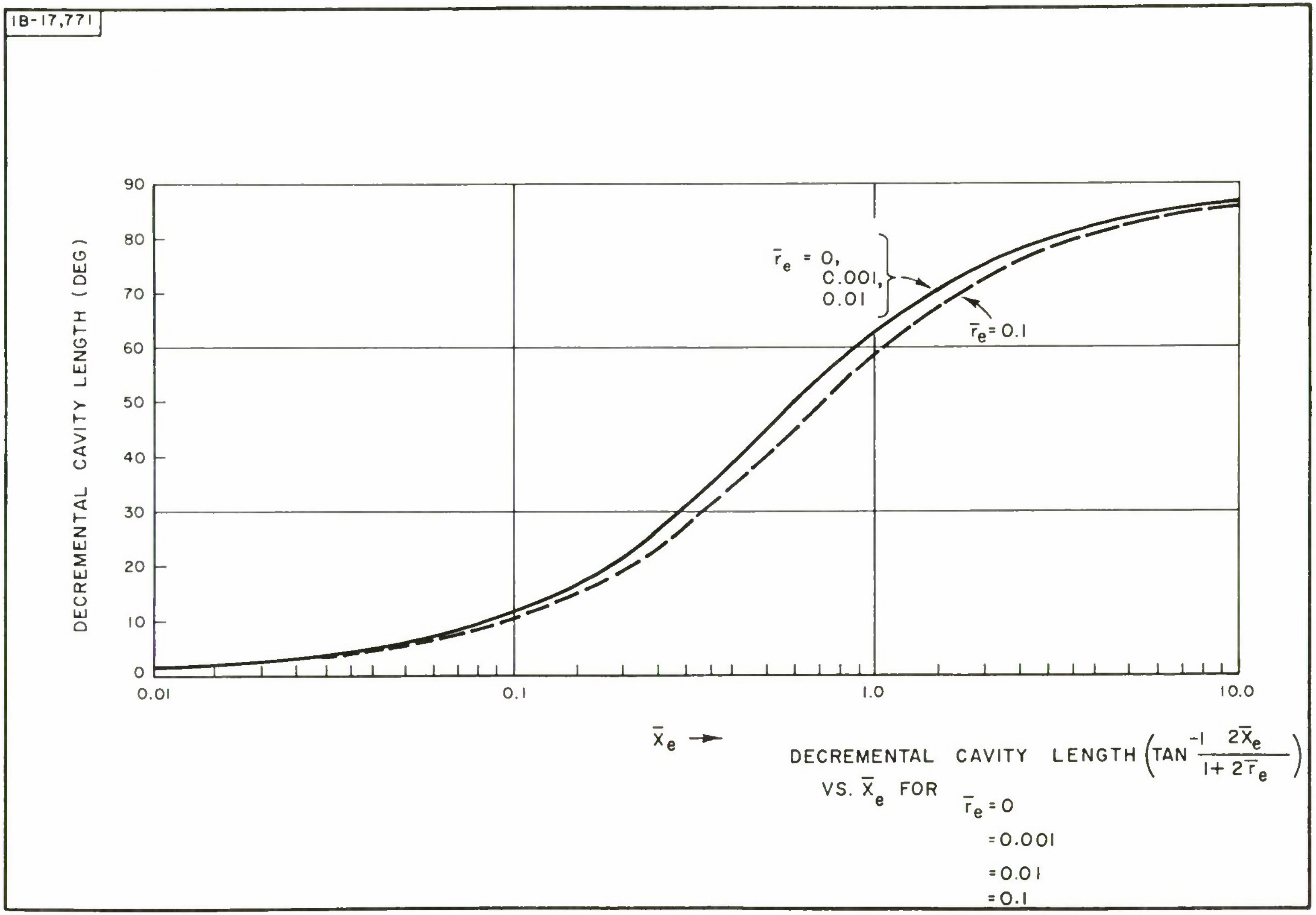

Figure 4. Decremental Cavity Length

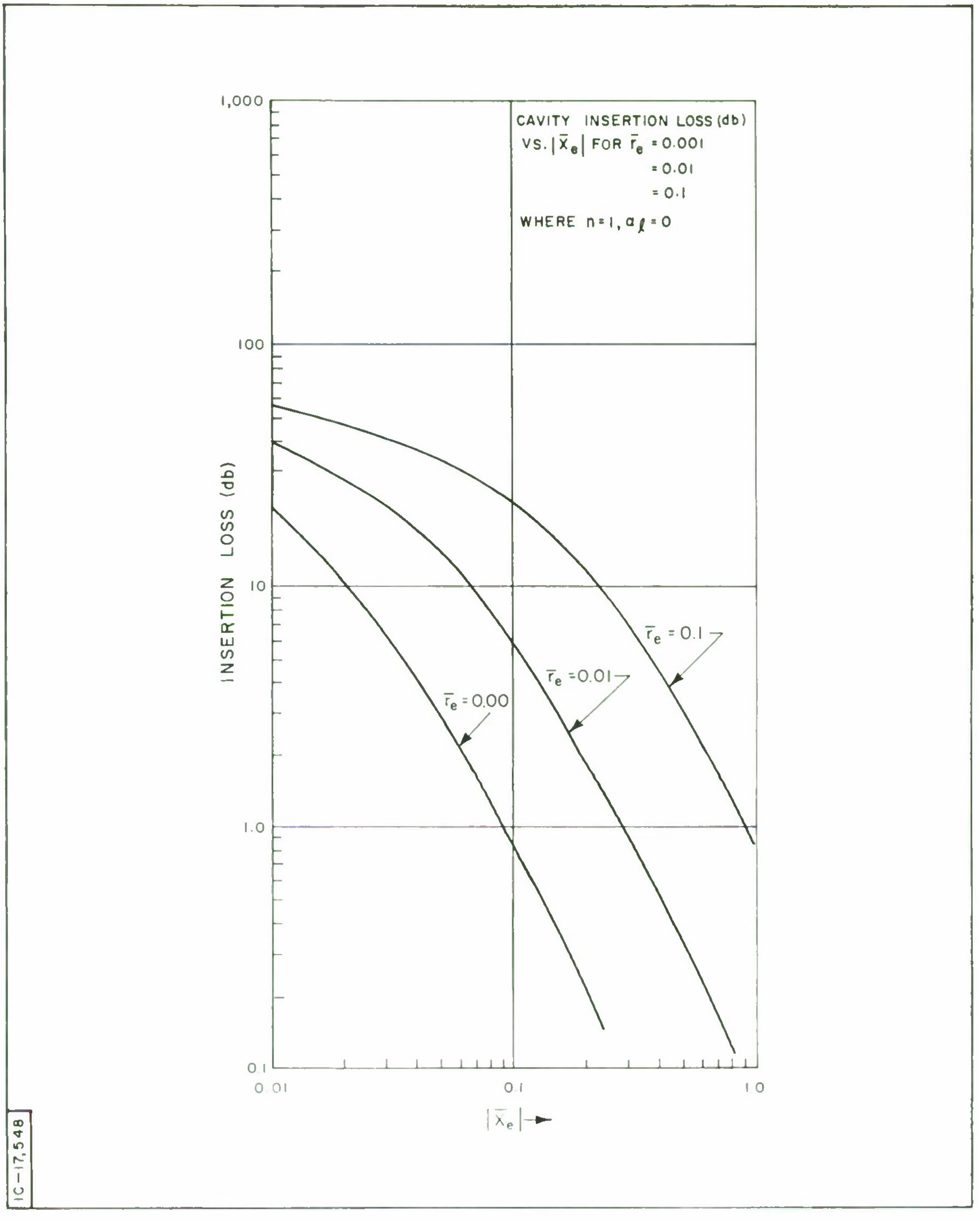

Figure 5. Cavity Insertion Loss

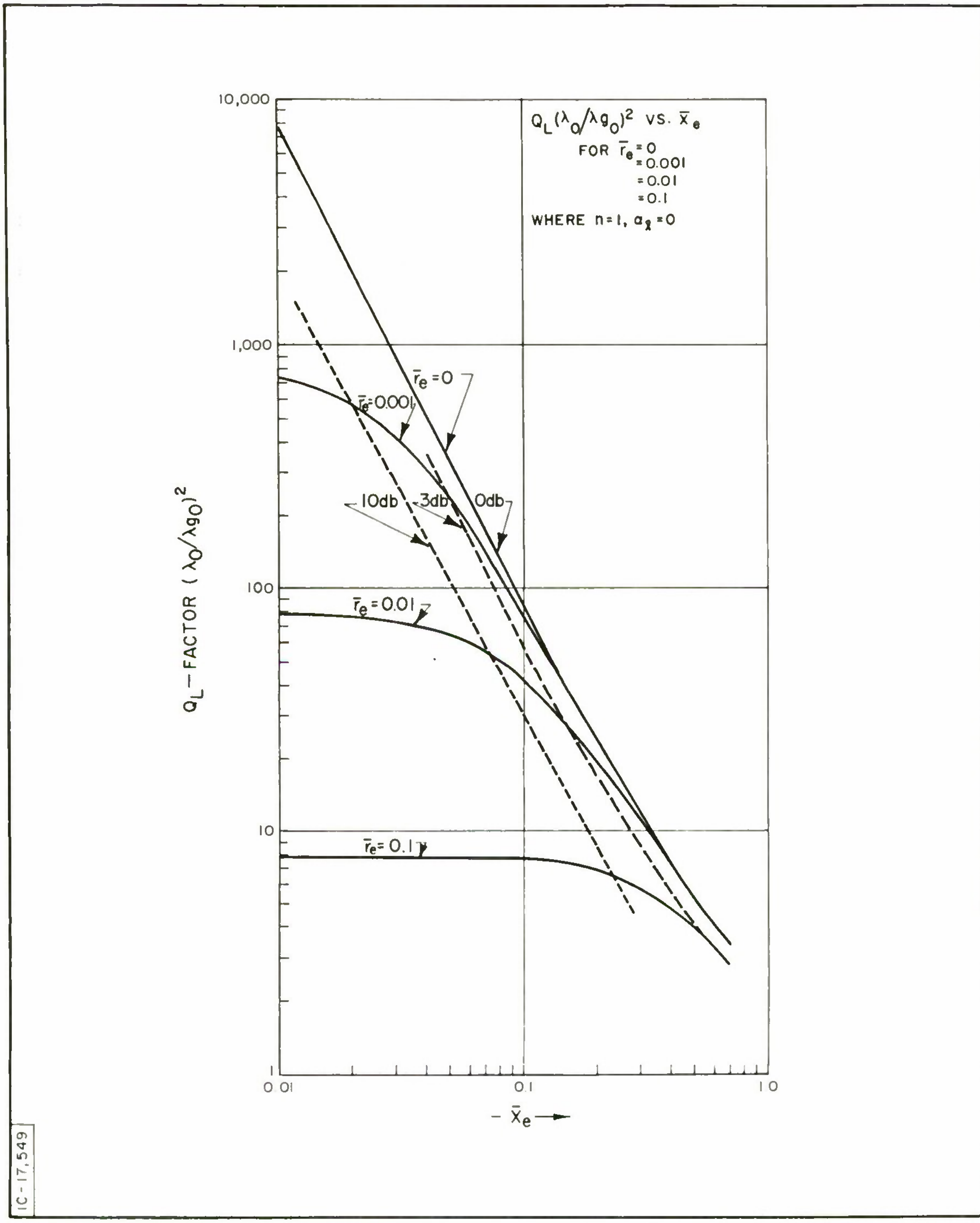

Figure 6. $Q_L\left(\lambda g_0/\lambda_0\right)^2$ versus $\bar{x}_e$

input-output losses only, for various values of $\bar{r}_e$. Since one is always interested in both Q_L and the insertion loss at resonance, lines of 0-db, 3-db, and 10-db insertion loss have been superimposed on the plot. The data have been obtained from Figure 5. Thus, for $\bar{r}_e = 0.001$ and an allowable 3-db insertion loss, $Q_L\left(\lambda_0/\lambda g_0\right)^2$ cannot exceed 230; or, for a desired $Q_L\left(\lambda_0/\lambda g_0\right)^2$ of 55 and a given value of $\bar{r}_e = 0.01$, the insertion loss will be 10 db, etc.

Although Figures 5 and 6 are for illustrative purpose only and will not accommodate all situations $(n > 1,\ \bar{x}_e > 0,\ \alpha_\ell \neq 0)$, Equations (23) and (37) always will.

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	Directorate of Radar & Optics, Electronic Systems Division, L. G. Hanscom Field, Bedford, Mass.

13. ABSTRACT The symmetrical—but lossy—waveguide transmission cavity is analyzed with respect to resonant length, minimum and maximum insertion losses, and 3-db frequencies (loaded Q-factor). Losses are considered to originate both in the cavity proper and in the input and output discontinuities. A preferred representation of those discontinuities makes it possible effectively to lump all the various losses together. All the quantities of interest (resonant length, insertion losses and Q_L-factor) are expressed in terms of the input and output discontinuity equivalent circuit. The results are quite general, but they will be of particular interest in the case of a transmission filter whose input and output are voltage-controlled varactors.

Normalized plots of resonant lengths, insertion loss at resonance, and Q_L-factors conclude the study. The reactive component of the input-output equivalent circuit is the independent variable. The resistive component is the parameter. Loci of constant insertion losses, plotted on the Q_L graphs, makes it possible to determine the maximum achievable Q_L for a given insertion loss and resistive component; or the maximum resistive component for a given Q_L and a prescribed insertion loss; or the minimum insertion loss for given Q_L and resistive component.

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Security Classification

14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
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